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Extremal Static AdS Black Hole/CFT Correspondence in Gauged Supergravities

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ABSTRACT

A recently proposed holographic duality allows the Bekenstein-Hawking entropy of extremal rotating black holes to be calculated microscopically, by applying the Cardy formula to the two-dimensional chiral CFTs associated with certain reparameterisations of azimuthal angular coordinates in the solutions. The central charges are proportional to the angular momenta of the black hole, and so the method degenerates in the case of static (non-rotating) black holes. We show that the method can be extended to encompass such charged static extremal AdS black holes by using consistent Kaluza-Klein sphere reduction ansätze to lift them to exact solutions in the low-energy limits of string theory or M-theory, where the electric charges become reinterpreted as angular momenta associated with internal rotations in the reduction sphere. We illustrate the procedure for the examples of extremal charged static AdS black holes in four, five, six and seven dimensions.

1 Introduction

The first computation of black hole entropy by counting microstates was performed for the case of five-dimensional extremal black holes [1]. This was subsequently extended to a variety of other examples, including black holes in four dimensions [2]. In this approach, the microscopic black hole states are equated with BPS D-brane states, and so it seemingly depends on features both of supersymmetry and of string theory.

On the other hand, it had already been known for quite some time that any consistent quantum theory of gravity on three-dimensional anti-de Sitter spacetime AdS_3 is equivalent to a two-dimensional conformal field theory, owing to the fact that the asymptotic symmetry group of AdS_3 is generated by (two copies of) the Virasoro algebra [3]. Combining the evaluation of the central charge with Cardy's formula for the asymptotic growth of states gives rise to a microscopic computation of entropy for black holes whose near-horizon geometry is locally AdS_3, such as the BTZ black hole. When the aforementioned extremal black holes in the dilute gas approximation [4] are embedded in one extra dimension, the near-horizon geometry becomes a direct product of a BTZ black hole and a sphere. The dilute gas approximation can be applied also to a rotating black hole [5]. This enables one to compute the statistical entropy without invoking supersymmetry or string theory [6].

Another holographic correspondence, namely AdS/CFT [7], has been extensively applied to a variety of black holes in gauged supergravity. The most concrete tests of AdS/CFT have been performed in cases for which the supergravity backgrounds are supersymmetric, although this duality may extend to non-supersymmetric backgrounds also. However, a *static* AdS black hole in the supersymmetric limit suffers from a naked singularity. This singularity can be avoided by adding rotation. Supersymmetric rotating AdS black holes in five dimensions were obtained in [8, 9, 10, 11], and subsequently, a boundary free-fermion approximation was used in order to obtain a microscopic evaluation of the entropy, up to a numerical factor of order unity [12].

Recently, a proposal was made for a new holographic duality between four-dimensional extremal Kerr black holes and a two-dimensional chiral conformal field theory [13], which extends the previous approaches of [14, 15, 16, 17, 18]. This proposed duality is motivated by the observation that the Cardy formula yields an entropy for the CFT which is in precise agreement with the Bekenstein-Hawking entropy of the extremal black hole. This proposed Kerr/CFT correspondence was subsequently extended to asymptotically flat and asymptotically AdS black holes with multiple angular momenta in diverse dimensions [19], where it was found that there is a chiral two-dimensional CFT associated with each independent rotation. Additional generalizations include Kaluza-Klein black holes [20], the Kerr-Newman-AdS black hole [21], NS5-branes [22], large classes of rotating black holes in gauged and ungauged supergravities [23], and several other types of rotating black holes [24, 25, 26, 27].

The calculation of the central charge is similar to the approach taken in [6], which is based on [3, 28]. Namely, one considers the asymptotic symmetry generators associated with a class of perturbations around the near-horizon Kerr geometry that obey suitably-chosen boundary conditions. This new method of computing microscopic entropy does not require the black hole to be supersymmetric or to be embedded within string theory, although it must be extremal. The rotation of the black hole plays a vital role, however, since the central charge is proportional to the angular momentum. In fact, the thermodynamic description breaks down for static black hole, in the sense that the central charge vanishes and the temperature diverges.

Since one can nonetheless use the Cardy formula to obtain the correct entropy for static black holes by taking a limit of rotating black holes, this suggests that there may be an alternative strictly static description that is not singular. In this paper we present such a description, based upon the observation that static charged black holes in many gauged supergravities can be lifted, by means of consistent Kaluza-Klein reduction formulae derived in [29], to become solutions in the ten or eleven-dimensional supergravities that arise as the low-energy limits of string theory or M-theory. The electric charges of the static black holes acquire the interpretation of rotations in the internal (spherical) dimensions, after the lifting has been performed.[1] The procedure developed in [13] can then be applied to the lifted solutions, with the Cardy formula for the entropies of the dual CFTs associated with the internal rotations giving a microscopic derivation of the Bekenstein-Hawking entropy of the original lower-dimensional extremal static AdS black hole. We apply this procedure to a variety of extremal static $U(1)$ charged AdS black holes in gauged supergravities in diverse dimensions. In particular, we consider five-dimensional 3-charge AdS black holes [30], four-dimensional 4-charge AdS black holes [31], seven-dimensional 2-charge AdS black holes [29] and six-dimensional single-charge AdS black holes [32]. We show that the microscopic

[1] A similar idea was discussed in [21], where it was argued that the extremal four-dimensional Kerr-Newman-AdS black hole could be viewed as a neutral five-dimensional configuration with a rotation in the fifth dimension. However, no consistent Kaluza-Klein reduction from five dimensions can give rise to the four-dimensional Einstein-Maxwell theory with cosmological constant, and so the four-dimensional Kerr-Newman-AdS black hole cannot be lifted to an actual neutral solution of any five-dimensional theory. Thus the example considered in [21] is perhaps somewhat heuristic in nature.

entropy matches perfectly with the Bekenstein-Hawking entropy for each of these cases.

2 Central charges of near-extremal metrics

In this section, we consider a large class of metrics that can be viewed as $\mathcal{M}$ bundles over AdS_2, where $\mathcal{M}$ can be any smooth manifold. The general form of the metrics is given by

$$\begin{aligned} ds^2 &= A\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + h_{\alpha\beta}\, dy^\alpha\, dy^\beta + \tilde{g}_{ij}\, \tilde{e}^i \tilde{e}^j\,, \\ \tilde{e}^i &= d\phi_i + k_i r\, d\tau\,, \end{aligned} \tag{2.1}$$

where A, $h_{\alpha\beta}$ and $\tilde{g}_{ij}$ can be functions of the coordinates y^α, and k_i's are constants. Let us assume that the index i runs from 1 to n. There exist n commuting diffeomorphisms that preserve the boundary structure at $r \to \infty$, namely

$$\zeta^i_m = -e^{-\mathrm{i}m\phi_i}\,\frac{\partial}{\partial\phi_i} - \mathrm{i}m\, r\, e^{-\mathrm{i}m\phi_i}\,\frac{\partial}{\partial r}\,, \qquad i = 1, \ldots n\,. \tag{2.2}$$

These diffeomorphisms generate n commuting Virasoro algebras. The central charges c_i in these Virasoro algebras, at the level of Dirac brackets of the associated charges $Q^i_{(n)} = 1/(8\pi)\int_{\partial\Sigma} k^i_{(n)}$, can be calculated in the manner described in [28], namely from the m^3 terms in the expressions

$$\frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta^i_{(m)}}[\mathcal{L}_{\zeta^i_{(-m)}} g, g] = -\frac{\mathrm{i}}{12}(m^3 + \alpha m)c_i\,, \tag{2.3}$$

where

$$\begin{aligned} k_\zeta[h,g] = \tfrac{1}{2}\Big[&\zeta_\nu\nabla_\mu h - \zeta_\nu\nabla_\sigma h_\mu{}^\sigma + \zeta_\sigma\nabla_\nu h_\mu{}^\sigma + \tfrac{1}{2}h\nabla_\nu\zeta_\mu - h_\nu{}^\sigma\nabla_\sigma\zeta_\mu \\ &+\tfrac{1}{2}h_{\nu\sigma}(\nabla_\mu\zeta^\sigma + \nabla_\sigma\zeta_\mu)\Big] *(dx^\mu \wedge dx^\nu)\,. \end{aligned} \tag{2.4}$$

Taking $g_{\mu\nu}$ to be given by (2.1), we find that the central charges are [2]

$$c_i = \frac{3k_i\mathcal{A}}{2\pi}\,, \qquad \mathcal{A} = \int\sqrt{h\,\tilde{g}}\, d^p y\int\prod_i d\phi_i\,, \tag{2.5}$$

where $\mathcal{A}$ is the volume of the manifold $\mathcal{M}$.

The structures of the near-horizon geometries of extremal black holes were extensively studied previously [33, 34], and were found to be encompassed within the general form of (2.1). It follows that the integrals in (2.5) can be identified with the entropies of the

[2]The ansatz (2.1) is slightly more general than the one originally presented in [23], in that the metric contribution $h_{\alpha\beta}\, dy^\alpha\, dy^\beta$ associated with the coordinates y^α is not restricted to being diagonal.

extremal black holes. In [23], it was shown by examining a wide class of rotating black holes that the constants k_i are given by

$$k_i = \frac{1}{2\pi T_i}\,, \tag{2.6}$$

where T_i is the associated Frolov-Thorne temperature on the horizon [35]. It follows, therefore, that the central charge is related to the Bekenstein-Hawking entropy S_{BH} by

$$c_i = \frac{6k_i\, S_{BH}}{\pi}\,. \tag{2.7}$$

This shows that any near-horizon geometry of the form (2.1) will have the property that the microscopic entropy for the i'th CFT associated with reparameterisations of ϕ_i, calculated using the Cardy formula

$$S_{BH} = \frac{\pi^2}{3}\, c_i\, T_i\,, \qquad \text{for each } i \tag{2.8}$$

will agree precisely with the Bekenstein-Hawking entropy S_{BH}.

It is perhaps worth emphasising at this point that the agreement between the microscopic calculation of the dual CFT entropy and the Bekenstein-Hawking calculation of the black hole entropy would break down if the constants k_i in (2.1) were equal to zero, which would be the case for static black holes. As can be seen from (2.6), the Frolov-Thorne temperature T_i would be infinite, while, from (2.7), the central charge c_i would vanish. One could still obtain the proper finite and non-zero result for the entropy of a static extremal black hole, using (2.8), by taking a static limit of rotating black holes. But if instead one starts form a black hole that is exactly static, then (2.8) cannot be used. This problem is circumvented by lifting the static black holes to higher dimensions, as we shall describe in the remainder of this paper.

3 Five-dimensional 3-charge AdS black holes

The maximal gauged supergravity in $D = 5$ has $SO(6)$ gauge symmetry. The Cartan subgroup is $U(1)^3$. The five-dimensional three-charge static AdS black hole solution was constructed in [30]. We adopt the convention of [29], and the solution is given by

$$\begin{aligned}
ds_5^2 &= -\mathcal{H}^{-2/3} f\, d\hat{t}^2 + \mathcal{H}^{1/3}(f^{-1}d\hat{r}^2 + \hat{r}^2 d\Omega_{3,\epsilon}^2)\,,\\
X_i &= H_i^{-1}\mathcal{H}^{1/3}\,, \qquad A^i_{(1)} = \Phi_i\, d\hat{t}\,, \qquad \Phi_i = -(1 - H_i^{-1})\alpha_i\,,\\
f &= \epsilon - \frac{\mu}{\hat{r}^2} + g^2\hat{r}^2\mathcal{H}\,, \qquad \mathcal{H} = H_1 H_2 H_3\,, \qquad H_i = 1 + \frac{\ell_i^2}{\hat{r}^2}\,,\\
\alpha_i &= \frac{\sqrt{1+\epsilon\sinh^2\beta_i}}{\sinh\beta_i}\,, \qquad \ell_i^2 = \mu\sinh^2\beta_i\,,
\end{aligned} \tag{3.1}$$

where $d\Omega_{3,\epsilon}^2$ is the unit metric for S^3, T^3 or H^3 for $\epsilon = 1, 0$ or -1, respectively. If all the charge parameters β_i are set equal, the solution becomes the five-dimensional Reissner-Nordström AdS black hole. The outer horizon is located at $\hat{r} = r_+$, which is the largest root of f. The temperature and entropy are given by

$$T_H = \frac{f'(r_+)}{4\pi\sqrt{\mathcal{H}(r_+)}}\,, \qquad S = \tfrac{1}{4} r_+^3\, \omega_{3,\epsilon} \sqrt{\mathcal{H}(r_+)}\,, \tag{3.2}$$

where $\omega_{3,\epsilon}$ is the volume for the $d\Omega_{3,\epsilon}^2$. The extremal limit is obtained when the function f has a double zero, $r = r_0$. This can be achieved by choosing parameters such that[3]

$$\mu = \frac{g^2}{r_0^2}\left(2\ell_{123}^2 + r_0^2 \sum_{i<j} \ell_{ij}^2 - r_0^6\right)\,, \qquad \epsilon = \frac{g^2}{r_0^4}\left(\ell_{123}^2 - r_0^4 \sum_{i<j} \ell_{ij}^2 - 2r_0^6\right)\,. \tag{3.3}$$

In this paper, we define $\ell_{i_1 \ldots i_n} = \ell_{i_1} \cdots \ell_{i_n}$. In this extremal limit, the temperature vanishes, but the entropy is non-vanishing, given by

$$S_0 = \frac{1}{4} r_0^3\, \omega_{3,\epsilon} \sqrt{\mathcal{H}_0}\,, \tag{3.4}$$

where $\mathcal{H}_0 \equiv \mathcal{H}(r_0)$. In the extremal limit, the near-horizon geometry of the black hole is the direct product $\mathrm{AdS}_2 \times S^3$. There exists a decoupling limit in which the near-horizon geometry becomes a solution in its own right. To see this, we note that in the near horizon, the function f can be expanded as

$$f = (\hat{r} - r_0)^2 V\,, \qquad V = \tfrac{1}{2} f''(r_0) = \frac{4g^2}{r_0^6}(\ell_{123}^2 + r_0^6)\,. \tag{3.5}$$

Making the coordinate transformation

$$\hat{r} = r_0(1 + \lambda\rho)\,, \qquad \hat{t} = \frac{\sqrt{\mathcal{H}_0}}{\lambda r_0 V} t\,, \tag{3.6}$$

and then sending the constant parameter $\lambda \to 0$, the solution becomes

$$\begin{aligned} ds_5^2 &= \frac{\mathcal{H}_0^{1/3}}{V}\left(-\rho^2 dt^2 + \frac{d\rho^2}{\rho^2}\right) + r_0^2 \mathcal{H}_0^{1/3} d\Omega_{3,\epsilon}^2\,, \\ X_i^0 &= \frac{\mathcal{H}_0^{1/3}}{H_i(r_0)}\,, \qquad A_{(1)}^i = \frac{k_i \rho}{g} dt\,, \end{aligned} \tag{3.7}$$

where the constant k_i is given by

$$k_i = \frac{1}{2\pi T_i}\,, \qquad T_i = -\frac{T_H'(r_0)}{g\Phi'(r_0)} = \frac{g(r_0^2 + \ell_i^2)^2(\ell_{123}^2 + r_0^6)}{\pi r_0^7 \alpha_i \ell_i^2 \sqrt{\mathcal{H}_0}}\,. \tag{3.8}$$

[3] It should be emphasised that the *extremal limit* is quite different from the *BPS limit*, which is obtained by sending μ to zero. Whilst the extremal limit is non-singular, the BPS limit has a naked singularity at $r = 0$. Similar remarks apply to the static AdS black holes in other dimensions that we discuss in subsequent sections.

Note that we have extracted the pure constant divergent terms of $A^i_{(1)}$ in (3.7) as pure gauge.

The metric (3.7) can be recast in terms of global AdS_2 coordinates (τ, r) rather than the Poincaré patch coordinates (t, ρ) by means of the transformations

$$\rho = r + \sqrt{1+r^2}\,\cos\tau\,, \qquad t = \frac{\sqrt{1+r^2}\,\sin\tau}{r + \sqrt{1+r^2}\,\cos\tau}\,. \tag{3.9}$$

After absorbing an exact form into the potential $A^i_{(1)}$ by means of a gauge transformation, the solution (3.7) becomes

$$\begin{aligned} ds_5^2 &= \frac{\mathcal{H}_0^{1/3}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + r_0^2\mathcal{H}_0^{1/3} d\Omega_{3,\epsilon}^2\,,\\ X_i^0 &= \frac{\mathcal{H}_0^{1/3}}{H_i(r_0)}\,, \qquad A^i_{(1)} = \frac{k_i r}{g} d\tau\,, \end{aligned} \tag{3.10}$$

We can now lift the solution back to $D = 10$, using the reduction ansatz given in [29], finding that the metric is given by

$$\begin{aligned} ds_{10}^2 &= \sqrt{\Delta} ds_5^2 + \frac{1}{g^2\sqrt{\Delta}} \sum_{i=1}^3 X_i^{-1}\left(d\mu_i^2 + \mu_i^2(d\hat{\phi} + g\,A^i_{(1)})^2\right)\,,\\ \Delta &= \mathcal{H}^{1/3}\sum_{i=1}^3 \frac{\mu_i^2}{H_i}\,, \qquad \sum_{i=1}^3 \mu_i^2 = 1\,. \end{aligned} \tag{3.11}$$

The near-horizon geometry of the black hole in the extremal limit is given by

$$\begin{aligned} ds_{10}^2 &= \frac{\sqrt{\Delta_0}\,\mathcal{H}_0^{1/3}}{V}\left[-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + V r_0^2\, d\Omega_{3,\epsilon}^2\right]\\ &\quad + \frac{1}{g^2\sqrt{\Delta_0}}\sum_{i=1}^3 (X_i^0)^{-1}\left(d\mu_i^2 + \mu_i^2(d\phi_i + k_i r\, d\tau)^2\right)\,, \end{aligned} \tag{3.12}$$

where $\Delta_0 = \Delta(r_0)$. The metric can be viewed as a warped $S^3 \times S^5$ bundle over AdS_2, with the fibre lying only in the S^5 directions. The volume of the warped $S^3 \times S^5$ is given by

$$\mathcal{A} = g^{-5}\, r_0^3 \sqrt{\mathcal{H}_0}\,\omega_{3,\epsilon}\omega_5\,. \tag{3.13}$$

where ω_5 is the volume for the unit S^5. The near-horizon metric (3.12) is clearly contained within the general ansatz (2.1); we may take

$$\begin{aligned} h_{\alpha\beta}\, dy^\alpha\, dy^\beta &= \sqrt{\Delta_0}\,\mathcal{H}_0^{1/3}\, r_0^2\, d\Omega_{3,\epsilon}^2 + \frac{1}{g^2\sqrt{\Delta_0}}\sum_{i=1}^3 (X_i^0)^{-1}\, d\mu_i^2\,,\\ \tilde{g}_{ij} &= \frac{\mu_i^2}{g^2\sqrt{\Delta_0}\,X_i^0}\,\delta_{ij}\,. \end{aligned} \tag{3.14}$$

It therefore follows from the general discussion given earlier that the central charge of the i'th Virasoro symmetry associated with reparameterisations of ϕ_i is given by

$$c_i = \frac{3k_i\mathcal{A}}{2\pi G_{10}} = \frac{6k_i S_0}{\pi}\,, \tag{3.15}$$

where S_0 is the $D=5$ black hole entropy given in (3.4). Here we have temporarily restored Newton's constant, which enters in the denominator of the Hawking entropy, $S=\mathcal{A}/(4G)$, and which we normally set to unity, in order to discuss the relation between the entropy in five dimensions and in ten dimensions. This follows by noting that the Kaluza-Klein reduction ansatz given in [29] implies that the Newton constants in ten and five dimensions are related by $G_{10}=g^{-5}\omega_5\, G_5$. Since the horizon areas are also related by $\mathcal{A}_{10}=g^{-5}\omega_5\,\mathcal{A}_5$, it follows that the ten-dimensional and five-dimensional entropies are equal. An analogous result holds in all the examples in other dimensions that we discuss in subsequent sections.

From the reduction ansatz (3.11), we see that the electric potential Φ_i is related to the angular velocities Ω_i of the azimuthal angles ϕ_i in $D=10$ by

$$\Omega_i = g\,\Phi_i\,. \tag{3.16}$$

It follows that T_i, given in (3.8), can be identified as the Frolov-Thorne temperature, and therefore that the entropy calculated using the Cardy formula (2.8), will agree precisely with the Bekenstein-Hawking entropy of the the extremal five-dimensional static AdS black hole (3.1).

It is of interest to note that we can perform a Kaluza-Klein reduction on the 3-manifold $d\Omega_{3\epsilon}^2$. The resulting solution becomes a rotating black hole in $D=7$, and the extremal black hole/CFT correspondence continues to hold.

4 Four-dimensional 4-charge AdS black holes

The maximum gauged supergravity in $D=4$ has $SO(8)$ gauge group, whose Cartan subgroup is $U(1)^4$. The four-charge static AdS black hole was constructed in [31, 36]

Following the convention of [29], the four-dimensional 4-charge AdS black hole solution is given by

$$\begin{aligned}
ds_4^2 &= -\mathcal{H}^{-1/2} f\, d\hat{t}^2 + \mathcal{H}^{1/2}(f^{-1}d\hat{r}^2 + \hat{r}^2 d\Omega_{2,\epsilon}^2)\,,\\
X_i &= H_i^{-1}\mathcal{H}^{1/4}\,, \qquad A_{(1)}^i = \Phi_i\, d\hat{t}\,, \qquad \Phi_i = -(1-H_i^{-1})\alpha_i\,,\\
f &= \epsilon - \frac{\mu}{\hat{r}} + 4g^2\hat{r}^2\mathcal{H}\,, \qquad \mathcal{H} = H_1H_2H_3H_4\,, \qquad H_i = 1+\frac{\ell_i}{\hat{r}}\,,
\end{aligned}$$

$$\alpha_i = \frac{\sqrt{1+\epsilon\sinh^2\beta_i}}{\sinh\beta_i}\,, \qquad \ell_i = \mu\sinh^2\beta_i\,, \tag{4.1}$$

where $d\Omega_{2,\epsilon}^2$ is the unit metric for S^2, T^2 or H^2 for $\epsilon = 1, 0$ or -1, respectively. If the charge parameters β_i are set equal, the solution becomes the standard Reissner-Nordström AdS black hole. The outer horizon is located at $\hat{r} = r_+$, which is the largest root of f. The temperature and entropy are given by

$$T_H = \frac{f'(r_+)}{4\pi\sqrt{\mathcal{H}(r_+)}}\,, \qquad S = \tfrac{1}{4} r_+^2\,\omega_{2,\epsilon}\,\sqrt{\mathcal{H}(r_+)}\,, \tag{4.2}$$

where $\omega_{2,\epsilon}$ is the volume for the $d\Omega_{2,\epsilon}^2$. The extremal limit is obtained when the function f has a double zero, $r = r_0$. This can be achieved by choosing parameters such that

$$\begin{aligned} \mu &= \frac{4g^2}{r_0}\left(2\ell_{1234} + r_0\sum_{i<j<k}\ell_{ijk} - r_0^3\sum_i \ell_i - 2r_0^4\right)\,,\\ \epsilon &= \frac{4g^2}{r_0^2}\left(\ell_{1234} - r_0^2\sum_{i<j}\ell_{ij} - 2r_0^3\sum_i\ell_i - 3r_0^4\right)\,. \end{aligned} \tag{4.3}$$

In this extremal limit, the temperature vanishes, but the entropy is non-vanishing, given by

$$S_0 = \frac{1}{4} r_0^2 \omega_{2,\epsilon}\,\sqrt{\mathcal{H}_0}\,, \tag{4.4}$$

where $\mathcal{H}_0 \equiv \mathcal{H}(r_0)$. In the extremal limit, the near-horizon geometry of the black hole is $\mathrm{AdS}_2 \times S^2$. There exists a decoupling limit that the near-horizon geometry is a solution on its own. To see this, we note that in the near horizon, the function f can be expanded as

$$f = (\hat{r} - r_0)^2 V\,, \qquad V = \tfrac{1}{2} f''(r_0) = \frac{4g^2}{r_0^4}\left(\ell_{1234} + r_0^3\sum_i \ell_i + 3r_0^4\right)\,. \tag{4.5}$$

Making the coordinate transformation (3.6) and then sending the constant parameter $\lambda \to 0$, the solution in global coordinates becomes

$$\begin{aligned} ds_4^2 &= \frac{\mathcal{H}_0^{1/2}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + r_0^2\mathcal{H}_0^{1/2} d\Omega_{2,\epsilon}^2\,,\\ X_i &= \frac{\mathcal{H}_0^{1/4}}{H_i(r_0)}\,, \qquad A_{(1)}^i = \frac{k_i r}{g} d\tau\,, \end{aligned} \tag{4.6}$$

where the constant k_i is given by

$$k_i = \frac{1}{2\pi T_i}\,, \qquad T_i = -\frac{T_H'(r_0)}{g\Phi_i'(r_0)}\,. \tag{4.7}$$

Using the reduction ansatz given in [29], we can now lift the solution back to $D = 11$. The metric is given by

$$ds_{11}^2 = \frac{\Delta_0^{2/3}\mathcal{H}_0^{1/2}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + V\,r_0^2 d\Omega_{2,\epsilon}^2\right)$$

$$
+\frac{1}{g^2\Delta_0^{1/3}}\sum_{i=1}^{4}\frac{1}{X_i^0}\left(d\mu_i^2+\mu_i^2(d\phi_i+k_i\,r d\tau)^2\right)\,,
$$

$$
\Delta_0=\sum_{i=1}^{4}X_i^0\mu_i^2\,,\qquad X_i^0=X_i(r_0)\,. \tag{4.8}
$$

Thus we see that the metric fits the general ansatz (2.1), and T_i can be identified as the Frolov-Thorne temperature. Following the same discussion in the previous section, the microscopic entropy for the i'th CFT associated with the reparameterisations of ϕ_i, calculated using the Cardy formula will agree precisely with the Bekenstein-Hawking entropy.

5 Seven-dimensional 2-charge AdS black holes

The maximal gauged supergravity in $D=7$ has $SO(5)$ gauge symmetry, whose Cartan subgroup is $U(1)^2$. The seven-dimensional 2-charge AdS black hole solution is given by [29]

$$
\begin{aligned}
ds_7^2 &= -\mathcal{H}^{-4/5}f\;d\hat{t}^2+\mathcal{H}^{1/5}(f^{-1}d\hat{r}^2+\hat{r}^2d\Omega_{5,\epsilon}^2)\,,\\
X_i &= H_i^{-1}\mathcal{H}^{2/5}\,,\qquad A_{(1)}^i=\Phi_i\,d\hat{t}\,,\qquad \Phi_i=-(1-H_i^{-1})\alpha_i\,,\\
f &= \epsilon-\frac{\mu}{\hat{r}^4}+\tfrac14 g^2\hat{r}^2\mathcal{H}\,,\qquad \mathcal{H}=H_1H_2\,,\qquad H_i=1+\frac{\ell_i^4}{\hat{r}^4}\,,\\
\alpha_i &= \frac{\sqrt{1+\epsilon\sinh^2\beta_i}}{\sinh\beta_i}\,,\qquad \ell_i^4=\mu\sinh^2\beta_i\,,
\end{aligned} \tag{5.1}
$$

where $d\Omega_{5,\epsilon}^2$ is the unit metric for S^5, T^5 or H^5 for $\epsilon=1,0$ or -1, respectively. The horizon is at $\hat{r}=r_+$, which is the largest root of f. The temperature and entropy are given by

$$
T_H=\frac{f'(r_+)}{4\pi\sqrt{\mathcal{H}(r_+)}}\,,\qquad S=\tfrac14 r_+^5\,\omega_{5,\epsilon}\sqrt{\mathcal{H}(r_+)}\,, \tag{5.2}
$$

where $\omega_{5,\epsilon}$ is the volume for the $d\Omega_{5,\epsilon}^2$. The extremal limit is obtained when the function f has a double zero, $r=r_0$. This can be achieved by choosing parameters such that

$$
\mu=\frac{g^2}{8r_0^2}\left(3\ell_{12}^4+r_0^4(\ell_1^4+\ell_2^4)-r_0^8\right)\,,\qquad \epsilon=-\frac{g^2}{8r_0^6}\left(\ell_{12}^4-r_0^4(\ell_1^4+\ell_2^4)-3r_0^8\right)\,. \tag{5.3}
$$

In this extremal limit, the temperature vanishes, but the entropy is non-vanishing, given by

$$
S_0=\tfrac14 r_0^5\omega_{5,\epsilon}\sqrt{\mathcal{H}_0}\,, \tag{5.4}
$$

where $\mathcal{H}_0\equiv\mathcal{H}(r_0)$. In the extremal limit, the near-horizon geometry of the black hole is $\mathrm{AdS}_2\times S^5$. There exists a decoupling limit that the near-horizon geometry is a solution on its own. To see this, we note that in the near horizon, the function f can be expanded as

$$
f=(\hat{r}-r_0)^2V\,,\qquad V=\tfrac12 f''(r_0)=\frac{g^2}{2r_0^8}(3\ell_{12}^4-r_0^4(\ell_1^4+\ell_2^4)+3r_0^8)\,. \tag{5.5}
$$

Making the coordinate transformation (3.6) and then sending the constant parameter $\lambda \to 0$, the solution in AdS_2 global coordinates becomes

$$
\begin{aligned}
ds_5^2 &= \frac{\mathcal{H}_0^{2/5}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + r_0^2\mathcal{H}_0^{2/5}d\Omega_{5,\epsilon}^2\,,\\
X_i &= \frac{\mathcal{H}_0^{2/5}}{H_i(r_0)}\,, \qquad A_{(1)}^i = \frac{k_i r}{g}d\tau\,,
\end{aligned}
\tag{5.6}
$$

where the constant k_i is given by

$$
k_i = \frac{1}{2\pi T_i}\,, \qquad T_i = -\frac{T_H'(r_0)}{g\Phi_i'(r_0)}\,. \tag{5.7}
$$

Again we can use the reduction ansatz in [29] to lift the solution back to $D = 11$. The metric is given by

$$
\begin{aligned}
ds_{11}^2 &= \frac{\Delta_0^{1/3}\mathcal{H}_0^{2/5}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + V\, r_0^2 d\Omega_{5,\epsilon}^2\right)\\
&\quad + \frac{1}{g^2\Delta_0^{2/3}}\left(\frac{1}{X_0^0}d\mu_0^2 + \sum_{i=1}^{2}\frac{1}{X_i^0}(d\mu_i^2 + k_i r\, d\tau)^2\right)\,,\\
\Delta_0 &= \sum_{\alpha=0}^{2} X_\alpha^0\mu_\alpha^2\,, \qquad X_i^0 = X_i(r_0)\,, \qquad X_0^0 = (X_1^0X_2^0)^{-2}\,.
\end{aligned}
\tag{5.8}
$$

Thus we see that the metric fits the general ansatz (2.1), and T_i can be identified as the Frolov-Thorne temperature. Following the same discussion as in the previous sections, the microscopic entropy for the i'th CFT associated with the reparameterisations of ϕ_i, calculated using the Cardy formula will agree precisely with the Bekenstein-Hawking entropy.

6 Six-dimensional single-charge AdS black holes

The gauged supergravity in $D = 6$ constructed in [37] has a $SU(2)$ gauge symmetry. The $U(1)$ charged AdS black hole was constructed in [32]. The solution is given by

$$
\begin{aligned}
ds_6^2 &= -H^{-3/2}f\,d\hat{t}^2 + H^{1/2}(f^{-1}d\hat{r}^2 + \hat{r}^2 d\Omega_{4,\epsilon}^2)\,,\\
X &= H^{-1/4}\,, \qquad A_{(1)} = \Phi\, d\hat{t}\,, \qquad \Phi = -\sqrt{2}(1-H^{-1})\alpha\, d\hat{t}\,,\\
f &= \epsilon - \frac{\mu}{\hat{r}^3} + \tfrac{2}{9}g^2\hat{r}^2H^2\,, \qquad H = 1 + \frac{\ell^3}{\hat{r}^3}\,,\\
\alpha &= \frac{\sqrt{1+\epsilon\sinh^2\beta}}{\sinh\beta}\,, \qquad \ell^3 = \mu\sinh^2\beta\,.
\end{aligned}
\tag{6.1}
$$

The temperature and the entropy are given by

$$
T_H = \frac{f'(r_+)}{4\pi H(r_+)}\,, \qquad S = \tfrac{1}{4}r_+^4\, H(r_+)\,\omega_{4,\epsilon}\,, \tag{6.2}
$$

where r_+ is the largest root of f. The solution becomes extremal when the parameters satisfy

$$\mu = \frac{4g^2(2\ell^6 + \ell^3 r_0^3 - r_0^6)}{27 r_0}\,, \qquad \epsilon = \frac{2g^2(\ell^6 - 4\ell^3 r_0^3 - 5r_0^6)}{27 r_0^4}\,. \tag{6.3}$$

In this case, the function f has a double zero at $\hat{r} = r_0$, and its expansion near $\hat{r} = r_0$ is given by

$$f = (\hat{r} - r_0)^2 V\,, \qquad V = \tfrac{1}{2} f''(r_0) = \frac{2g^2(2\ell^6 - 2\ell^3 r_0^3 + 5r_0^6)}{9 r_0^6}\,. \tag{6.4}$$

Using the same method as in the previous examples, we obtain the near-horizon solution, written in AdS_2 global coordinates, given by

$$\begin{aligned} ds_6^2 &= \frac{H_0^{1/2}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2}\right) + r_0^2 H_0^{1/2} r_0^2 d\Omega_{4,\epsilon}^2\,, \\ X_i &= X_i^0 = H_0^{-1/4}\,, \qquad A_{(1)} = \frac{kr}{g} d\tau\,, \end{aligned} \tag{6.5}$$

where

$$k = \frac{1}{2\pi T}\,, \qquad T = -\frac{T_H'(r_0)}{g\Phi'(r_0)}\,. \tag{6.6}$$

It was shown in [32] that the six-dimensional gauged supergravity can be obtained by Kaluza-Klein reduction from the massive type IIA theory [38] in $D = 10$. The reduction ansatz has a singular warp factor, associated with the D4/D8 system. However, the metric in the D4-brane frame is regular [39]. Following the reduction ansatz in [32], we have

$$\begin{aligned} ds_{10}^2 &= \frac{\Delta_0^{2/5} H_0^{1/2}}{V}\left(-(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + V r_0^2 d\Omega_{4,\epsilon}^2\right) \\ &\quad + \frac{\Delta_0^{2/5} X_0^2}{2g^2} d\xi^2 + \frac{\Delta_0^{3/5}}{2g^2 \Xi_0}\cos^2\xi\left(d\theta^2 + \sin^2\theta d\phi^2 + (\sigma_3 + 2\,k\,r d\tau)^2\right)\,, \\ \Delta_0 &= X_0 \cos^2\xi + X_0^{-3}\sin^2\xi\,, \qquad X_0 = X(r_0)\,, \\ \sigma_3 &= d\psi + \cos\theta\, d\phi\,. \end{aligned} \tag{6.7}$$

Thus we see that the metric fits the general ansatz (2.1), and T can be identified as the Frolov-Thorne temperature. Following the same discussion in the previous sections, the microscopic entropy for the CFT associated with the reparameterisations of ψ, calculated using the Cardy formula, will agree precisely with the Bekenstein-Hawking entropy.

7 Conclusions

In this paper, we have extended the extremal black hole/CFT correspondence to static $U(1)$ charged black holes in gauged supergravities in a variety of dimensions. The first step was to use a limiting procedure to obtain a near-horizon geometry of the form $\mathrm{AdS}_2 \times S^n$. We then lifted these geometries up to ten or eleven dimensions on S^m, where they all have the form of a warped $S^n \times S^m$ bundle over AdS_2. The reparameterisations of the azimuthal directions of S^m, which are fibred over the AdS_2, are associated with Virasoro symmetries and the central charges of the corresponding two-dimensional chiral CFTs can be read off. All of the higher-dimensional geometries have the form of a manifold bundle over AdS_2, which we have shown implies that the microscopic entropy of each CFT, calculated using the Cardy formula, will agree precisely with the Bekenstein-Hawking entropy.

There are two seemingly distinct holographic dualities that can be applied to the above backgrounds, namely, the AdS/CFT correspondence and the extremal black hole/CFT correspondence. Before the near-horizon limit is taken, from the AdS/CFT perspective the asymptotically locally AdS geometry corresponds to a conformal fixed point in the UV region of a Yang-Mills theory at finite temperature. The charges of the AdS black hole are associated with the presence of chemical potentials [40]. Coming in towards the black hole corresponds to the field theory undergoing a Renormalization Group flow. Thus, the near-horizon region corresponds to the low-energy limit of the Yang-Mills theory. At the same time, from the extremal black hole/CFT perspective, the near-horizon region corresponds to a set of two-dimensional CFTs, whose central charges are related to the charges of the AdS black hole. Via these two types of holographic dualities, the low-energy limit of the Yang-Mills theory is described by the two-dimensional CFTs. It would be interesting if this link of the triality could be made direct. However, since the near-horizon geometry contains an AdS_2 component, this indicates that the $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence might apply, which is still not well understood. Thus, perhaps one type of holographic correspondence could help shed light on the other.

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